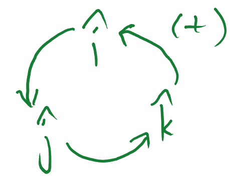
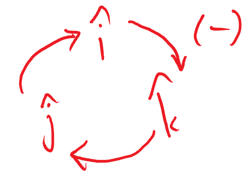
cross-products of $\hat{i}, \hat{j}, \hat{k}$ unit (basis) vectors

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

$$\left.\begin{aligned}\hat{i} \times \hat{j} &= \hat{k}\\ \hat{j} \times \hat{k} &= \hat{i}\\ \hat{k} \times \hat{i} &= \hat{j}\end{aligned}\right\}\text{Notice order}$$





$\hat{j} \times \hat{j} = \underset{\sim}{0}$

$|\hat{j}| \cdot |\hat{j}| \cdot \cancelto{0}{\sin\theta} = 0$

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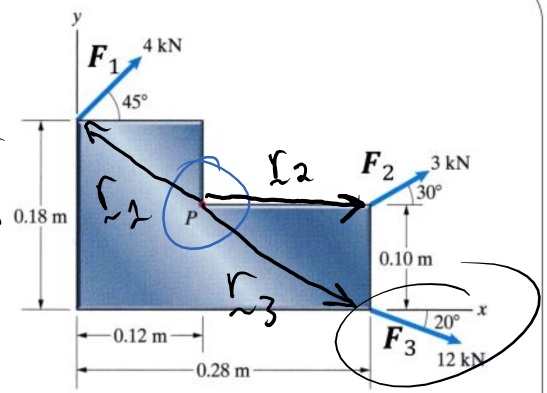
Three forces act on the plate. Determine the sum of the moments of the three forces about point *P*.



$$\underset{\sim}{M}_P = \underset{\sim}{r}_1 \times \underset{\sim}{F}_1 + \underset{\sim}{r}_2 \times \underset{\sim}{F}_2 + \underset{\sim}{r}_3 \times \underset{\sim}{F}_3$$

$$\underset{\sim}{r}_1 = (-0.12\,\hat{i} + 0.08\,\hat{j})\,m$$

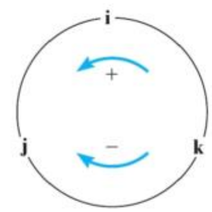
$$\underset{\sim}{r}_2 = 0.16\,\hat{i}\ m$$

$$\underset{\sim}{r}_3 = (0.16\,\hat{i} - 0.10\,\hat{j})\,m$$

$$\underset{\sim}{F}_1 = 4kN\,(\cos 45^\circ\,\hat{i} + \sin 45^\circ\,\hat{j})$$

$$\underset{\sim}{F}_2 = 3kN\cdot(\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j})$$

$$\underset{\sim}{F}_3 = 12kN\cdot(\cos 20^\circ\,\hat{i} - \sin 20^\circ\,\hat{j})$$



$$\underset{\sim}{r}_1 \times \underset{\sim}{F}_1 = (-0.12\,\hat{i} + 0.08\,\hat{j}) \times (\cos 45^\circ\,\hat{i} + \sin 45^\circ\,\hat{j})\cdot(4\,kN\cdot m)$$

$$= (-0.12\cdot\cos 45^\circ)\,\underbrace{\hat{i}\times\hat{i}}_{\underset{\sim}{0}} + (-0.12\cdot\sin 45^\circ)\,\underbrace{\hat{i}\times\hat{j}}_{\hat{k}}$$

$$+ (0.08\cdot\cos 45^\circ)\,\underbrace{\hat{j}\times\hat{i}}_{-\hat{k}} + (0.08\cdot\sin 45^\circ)\,\underbrace{\hat{j}\times\hat{j}}_{\underset{\sim}{0}}$$

and so on...

$$\underset{\sim}{M}_P = 0.145\,\hat{k}\ \text{kN}\cdot\text{m}$$

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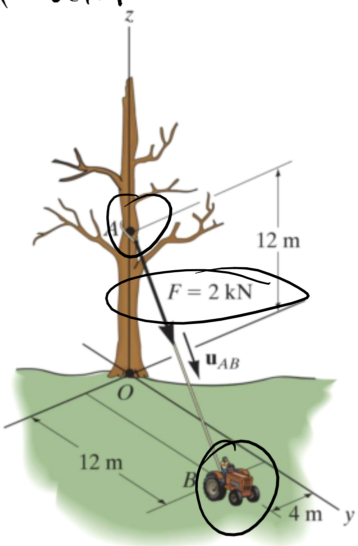
Determine the moment produced by the force **F** about ~~point **O**~~. x-axis



$$\underset{\sim}{F} = F \cdot \hat{u}_{AB} \qquad \hat{u}_{AB} = \frac{\underset{\sim}{r}_{AB}}{|r_{AB}|}$$

$$\underset{\sim}{r}_{AB} = \underset{\sim}{r}_B - \underset{\sim}{r}_A$$

$$= 4m\,\hat{i} + 12m\,\hat{j} - 12m\,\hat{k}$$

$$\hat{u}_{AB} = \frac{4\hat{i} + 12\hat{j} - 12\hat{k}}{\sqrt{4^2 + 144 + 144}}$$

$$\underset{\sim}{F} = (2kN)\,\frac{4\hat{i} + 12\hat{j} - 12\hat{k}}{\sqrt{304}}$$

$$\simeq (0.4588\,\hat{i} + 1.376\,\hat{j} - 1.376\,\hat{k})\,kN$$

$$\underset{\sim}{M}_o = \underset{\sim}{r}_A \times \underset{\sim}{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix} kN \cdot m$$

$$= [\,\hat{i}\,(0 \times -1.376 - 12 \cdot 1.376)$$

$$-\hat{j}\cdot\left(0\times\cancel{1.376}-0.4588\cdot 12\right)$$

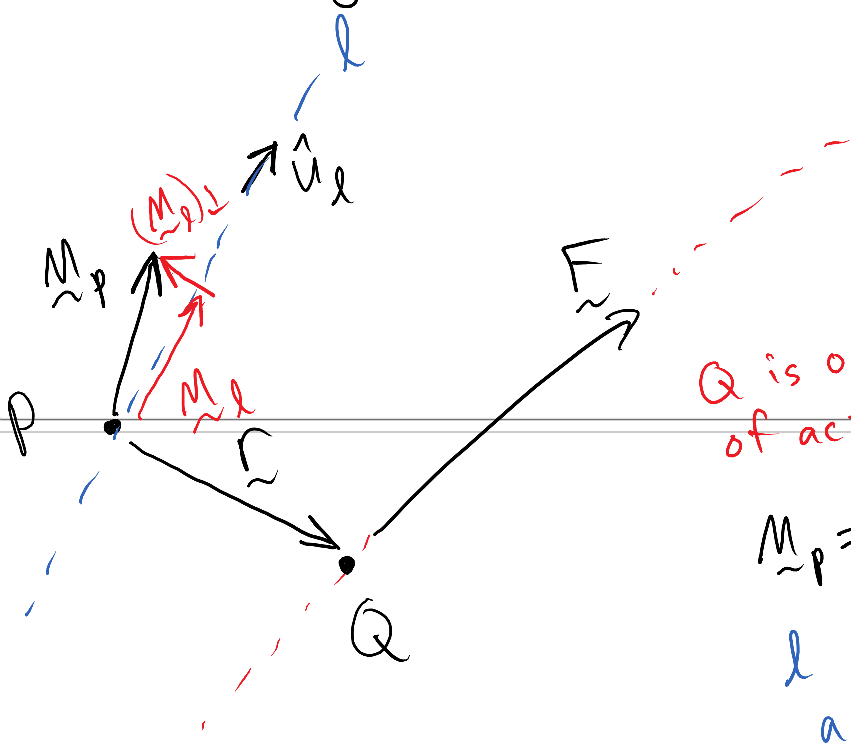
$$\left.+\hat{k}\cdot\left(\underbrace{0\cdot\cancel{1.376}}_{0}-\underbrace{0\cdot\cancel{0.4588}}_{0}\right)\right]\,kN\cdot m$$

$$=\left(-16.5\,\hat{i}+5.51\,\hat{j}\right)kN\cdot m$$

Moment of a force about specified axis

Moments are vectors

∴ they can be resolved into components <u>parallel</u> and <u>perpendicular</u> to any given direction



Q is on line of action of F

$$M_p = r \times F$$

ℓ is line of action of M_ℓ

$(M_\ell)_\perp$ is $\perp$ to M_ℓ

The projection of M_p along ($\parallel$ to) line ℓ is found using the dot product: $M_\ell = M_p \cdot \hat{u}_\ell$

$$= (r \times F) \cdot \hat{u}_\ell$$

$$\underset{\sim}{M}_{\ell} = M_{\ell} \cdot \hat{u}_{\ell} = (\underset{\sim}{M}_{p} \cdot \hat{u}_{\ell}) \cdot \hat{u}_{\ell}$$

The $\perp$ component: $(\underset{\sim}{M}_{\ell})_{\perp} = \underset{\sim}{M}_{p} - \underset{\sim}{M}_{\ell}$

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I>Clicker questions:

1) If $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$, then what will be the value of $\boldsymbol{M} \bullet \boldsymbol{r}$?

A) 0 B) 1

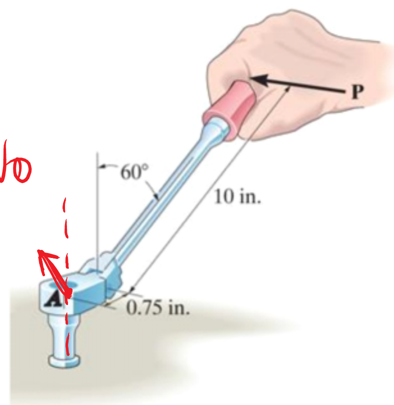
C) r^2 F D) None of the above.

$\underset{\sim}{r} \times \underset{\sim}{F}$ $\underset{\sim}{r}$ & $\underset{\sim}{F}$ form a plane (if $\theta \neq 0$)

$\underset{\sim}{M}$ is $\perp$ to that plane

2) With the force $\boldsymbol{P}$, a person is creating a moment $\boldsymbol{M}_A$ using this flex-handle socket wrench. Does all of $\boldsymbol{M}_A$ act to turn the socket?

A) YES B) NO



What if: A) Yes B) No

[redacted]

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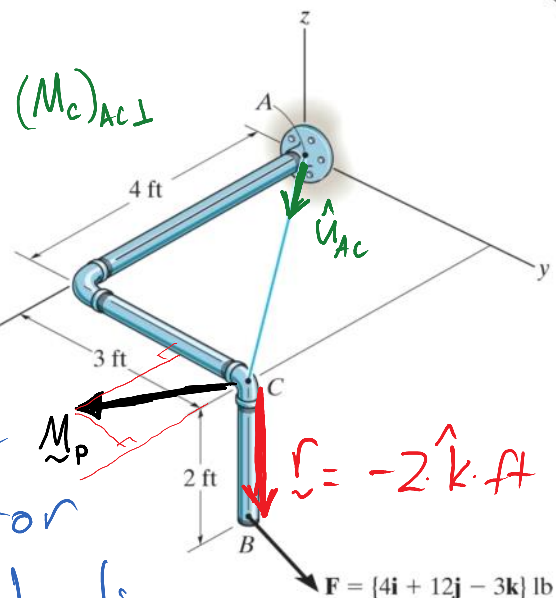
Determine the moment of the force F about the axis extending between A and C.

→ that is, find $(M_c)_{AC\perp}$



Since the axis we are concerned about is from A to C, we can pick a position vector from any point on that line.

$\therefore \underset{\sim}{r} = -2\hat{k}\cdot ft$

$$\underset{\sim}{M}_c = (-2\hat{k}\,ft) \times (4\hat{i} + 12\hat{j} - 3\hat{k})\,lb$$

$$= (\underbrace{-8\hat{k}\times\hat{i}}_{\hat{j}} - \underbrace{24\hat{k}\times\hat{j}}_{-\hat{i}} + \underbrace{6\hat{k}\times\hat{k}}_{\underset{\sim}{0}})\,lb\text{-}ft$$

$$= (24\hat{i} - 8\hat{j})\,lb\text{-}ft$$

Unit vector along AC :

$$\hat{u}_{AC} = \frac{4\hat{i} + 3\hat{j}}{\sqrt{4^2+9^2}} = \frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$$

$$(\underset{\sim}{M}_C)_{AC} = (\underset{\sim}{M}_C \bullet \hat{u}_{AC})\, \hat{u}_{AC}$$

$$= \left(24 \times \frac{4}{5} - 8 \times \frac{3}{5}\right)\left(\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right) \text{ lb}\cdot\text{ft}$$

$$= 14.4 \times \left(\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right) \text{ lb}\cdot\text{ft}$$

$$= (11.52\,\hat{i} + 8.64\,\hat{j}) \text{ lb}\cdot\text{ft}$$

this is the component of $\underset{\sim}{M}_c$ that is parallel to AC

$$(\underset{\sim}{M}_c)_{AC\perp} = \underset{\sim}{M}_c - (\underset{\sim}{M}_c)_{AC}$$

$$= \left[(24\hat{i} - 8\hat{j}) - (11.52\,\hat{i} + 8.64\,\hat{j})\right] \text{ lb}\cdot\text{ft}$$

$$= (12.48\,\hat{i} - 16.64\,\hat{j}) \text{ lb}\cdot\text{ft}$$

this is the component of $\underset{\sim}{M}_c$ that is perpendicular to AC

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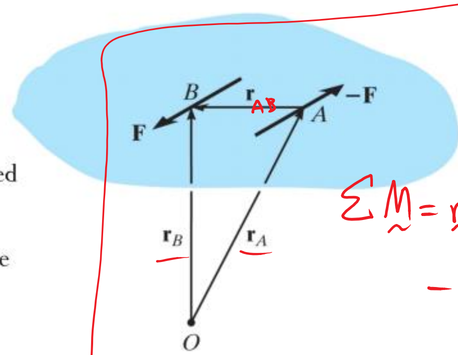
Moment of a couple

A **couple** is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d.

Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction.

The moment produced by a couple is called **couple moment**.

Let's determine the sum of the moments of both couple forces about **any** arbitrary point:



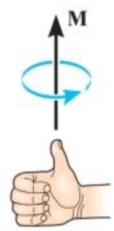
$\Sigma \underset{\sim}{M} = \underset{\sim}{r}_A \times -\underset{\sim}{F} + \underset{\sim}{r}_B \times \underset{\sim}{F}$

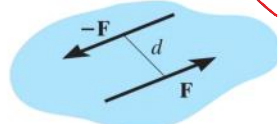
$= (-\underset{\sim}{r}_A + \underset{\sim}{r}_B) \times \underset{\sim}{F}$

$= \quad - \underset{\sim}{r}_A) \times \underset{\sim}{F}$

$= \underset{\sim}{r}_{AB} \times \underset{\sim}{F}$

Couple moment is a free vector.



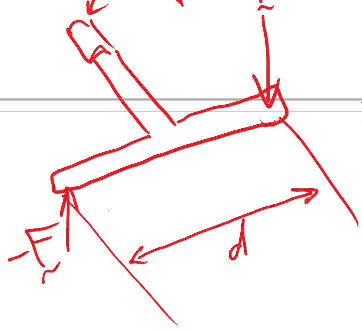


T-bar

lug wrench

lug nut feels $\underset{\sim}{F}$

$\Sigma F = 0$



$\Sigma M = F \cdot d$